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An analytic radiative transfer model for a coupled atmosphere and leaf canopy

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Abstract. A new analytical radiative transfer model of a leaf canopy is developed that approximates multiple-scattering radiance by a four-stream formulation. The canopy model is coupled to a homogeneous atmospheric model as well as a non-Lambertian lower boundary soil surface. The same four-stream formulation is also used for the calculation of multiple scattering in the atmosphere. Comparisons of radiance derived from the four-stream model with those calculated by an iterative numerical solution of the radiative transfer equation show that the analytic model has a very high accuracy, even with a turbid atmosphere and a very dense canopy in which multiple scattering dominates. Because the coupling of canopy and atmospheric models fully accommodates anisotropic surface reflectance and atmospheric scattering and its effect on directional radiance, the model is especially useful for application to directional radiance and measurements obtained by remote sensing. Retrieval of biophysical parameters using this model is under investigation.

1. Introduction

Modeling the radiation field emergent from a leaf canopy in the solar reflective spectrum has recently attracted increasing attention [*Goel*, 1988; *Myneni et al.*, 1990a; *Strahler*, 1994]. With a new generation of instruments capable of making multiangle measurements of this radiation field through the atmosphere, the opportunity arises to invert canopy models to yield remote estimations of such parameters as leaf area index (LAI), leaf angle distribution (LAD), and other quantities of interest to studies of the photosynthetic behavior of vegetation covers and their surface energy balances. These instruments include airborne sensors, such as the advanced solid-state array spectroradiometer (ASAS) [*Irons et al.*, 1991], and planned spaceborne sensors, such as the multiangle imaging spectroradiometer (MISR) [*Diner et al.*, 1989]. A number of analytic canopy bidirectional reflectance models based on radiative transfer theory have been published [*Suits*, 1972; *Verhoef*, 1984; *Camillo*, 1987; *Nilson and Kuusk*, 1989; *Pinty et al.*, 1990; *Ahmad and Deering*, 1992], but there are two problems associated with these models. First of all, most are based on simple formulations or approximations of multiple-scattering radiance. For example, most models apply the two-stream approximation to calculate the multiple-scattering component. The two-stream approximation has been widely used for radiative flux calculations because of its simplicity, but its accuracy is quite limited when applied to model the angular characteristics of the canopy radiation field. In the near-infrared region, multiple scattering is over 50% of the total scattering in dense canopies [*Liang and Strahler*, 1993a], and here the accuracy of any model largely depends on the treatment of multiple-scattering radiance.

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Second, most analytic canopy models decouple the canopy from the atmosphere, since they focus mainly on the requirements of ground multiangle observations. For airborne and spaceborne sensors, the interaction between the atmosphere and canopy and the path radiance of the atmosphere cannot be neglected [*Gerstl and Zardecki*, 1985; *Liang and Strahler*, 1993a; *Rahman et al.*, 1993a, b]. Any change in canopy optical properties will affect sky radiance, since it is partly a function of the ground surface reflectance. As a result, variation in sky radiance will feed back to change the canopy reflectance and thus influence the inversion of biophysical parameters. Except for developing application-specific techniques, coupling the two media is a natural way to handle their radiative interaction.

In an earlier coupled atmosphere-surface model [*Liang and Strahler*, 1993b] we used an asymptotic fitting technique to approximate the multiple-scattering component of the canopy and incorporated the sky radiance distribution into that canopy model by means of the two-stream approximation. In the present study the four-stream approximation [*Liang and Strahler*, 1994a] is extended to calculate the multiply scattered radiance of the coupled medium, and, further, a non-Lambertian lower (soil) boundary is incorporated into this formulation.

In several canopy models [*Gerstl and Zardecki*, 1985; *Camillo*, 1987] the Henyey-Greenstein function has been used for the phase function of the leaf canopy. However, the asymmetry parameter was selected very arbitrarily. An empirical formula is developed in this study connecting the asymmetry parameter to leaf biophysical parameters from the case of the spherical canopy. Since the real scattering phase function is still used for single scattering, this approximation does not result in severe errors for some extreme leaf angle distributions. Thus all parameters for the leaf canopy have very clear physical meanings. One important advantage of an analytical canopy reflectance model is that it offers the possibility of efficient retrieval of canopy (and atmospheric) parameters through inversion. In this process an inversion procedure (e.g., the *Powell* [1964] algorithm as modified by *Brent* [1973] and *Zhangwill* [1967]) is applied to observed data. The procedure makes iterative computations of the model, searching the parameter space until the observations are well-fit. The values of the parameters thus retrieved are then taken to be those of the canopy and/or atmosphere. The accuracy of the retrievals depends on a number of factors, including numerical considerations and how well the model represents the true physics of the atmosphere and canopy. Inversion of physical models for remote sensing applications is discussed more fully by *Pinty and Verstraete* [1992].

Inversion and validation of parameter retrievals requires that suites of measurements of radiances and physical parameters be made under varying conditions. At present, we have not carried out such inversions, as field measurements are required. However, we hope to do so in the future. In this paper we present a coupled reflectance model and compare its results with those of a more complex, coupled radiative transfer model that solves the radiative transfer equation with high accuracy. Although these comparisons do not constitute a proper validation in the sense of *Pinty and Verstraete* [1992], they demonstrate that the four-stream approximation for multiple scattering is quite accurate and thus lay the foundation for rapid iterative inversion and retrieval of biophysical canopy parameters from remotely sensed observations.

The rest of the paper is organized as follows: Section 2 introduces the radiative transfer equations for both atmosphere and canopy. The four-stream formulation for multiple scattering is given in section 3. Section 4 discusses model validation, and a brief discussion and conclusion will be given in the final section.

2. Radiative Transfer Equations for the Coupled Atmosphere and Canopy

Although we treat the radiation field of the atmosphere and canopy as a single coupled medium, the radiative transfer models of atmosphere and canopy will be separately described because of their different attenuating properties. In the present model the optical depth τ replaces geometric altitude z. The top of the atmosphere is set to $\tau = 0$, the bottom of the atmosphere is set to τ_a, and the total optical depth is τ_t. Therefore the optical depth of the canopy is $\tau_c = \tau_t - \tau_a$. For plant canopy this optical depth represents the amount of one-sided leaf area in a volume of canopy with unit surface area, which is often interpreted as the leaf area index (LAI). Both atmosphere and canopy are assumed to be horizontally infinite and homogeneous; thus one-dimensional radiative transfer equations considering only vertical variations are dealt with in this paper.

Atmospheric Radiative Transfer Model

For a plane-parallel homogeneous atmosphere in the absence of polarization the radiative transfer equation can be written as [*Lenoble*, 1985]

$$\mu \frac{\partial I(\tau, \Omega)}{\partial \tau} = I(\tau, \Omega) - \frac{\omega_a}{4\pi} \int_{4\pi} p(\Omega' \rightarrow \Omega) I(\tau, \Omega') \, d\Omega' \tag{1}$$

where ω_a is the single scattering albedo, $p(\Omega' \rightarrow \Omega)$ is the phase function, and $I(\tau, \Omega)$ is radiance in the direction Ω at optical depth τ. The quantity $\Omega(\mu, \phi)$ stands for an azimuthal angle ϕ and a zenith angle $\theta = \cos^{-1}(\mu)$.

The scattering properties of the atmosphere depend on molecular and aerosol particles. Thus the scattering phase function can be defined as a weighted average of individual scattering phase functions at specific scattering angles:

$$p(\Psi) = \frac{p_r(\Psi)\tau_r + p_a(\Psi)\tau_{ae}}{\tau_a}$$

with the constraint $(1/2) \int_0^\pi p(\Psi) \sin \Psi \, d\Psi = 1$. Here Ψ is the scattering angle dependent on solar zenith angle $\theta_0 = \cos^{-1} \mu_0$, viewing angle θ, and the angular difference between solar azimuth and viewing azimuth $\phi - \phi_0$. Two parameters τ_r and τ_{ae} are the molecular optical depth and aerosol optical depth, respectively, and $\tau_a = \tau_r + \tau_{ae}$. The one-term Henyey-Greenstein function is used as the aerosol phase function. For simplicity we assume in the following calculations that only molecules and aerosols are included in the atmosphere. Aerosols are treated as absorbing as well as scattering particles; for all cases we use a single scattering albedo $\omega = 0.92$.

To obtain a solution for (1), appropriate boundary conditions have to be specified. On the upper boundary the atmosphere is illuminated by a parallel beam in the direction (Ω_0) with net flux $i_0 = \pi F_0$, that is,

$$I(0, \Omega) = \delta(\Omega - \Omega_0) i_0$$

where $\mu < 0$, and $\delta(\Omega - \Omega_0)$ is the Dirac delta function with value unity when $\Omega = \Omega_0$ and zero when $\Omega \neq \Omega_0$. For the coupled medium the lower boundary condition at the bottom of the canopy will be discussed in the following section.

If we consider gaseous absorption in the atmosphere, i_0 should be replaced by $i_0 \exp[-\tau_g(1/|\mu_0| + 1/\mu)]$, where τ_g is the optical depth of the specific gas (e.g., water vapor or ozone).

According to these definitions, μ_0 must be negative. Since it is so often used in the following text, we will let $\mu_0 = |\mu_0|$ for simplicity.

Canopy Radiative Transfer Model

The one-dimensional radiative transfer equation of a flat homogeneous canopy is given by

$$-\mu \frac{\partial I(\tau, \Omega)}{\partial \tau} + h(\tau, \Omega, \Omega_0) G(\Omega) I(\tau, \Omega) = \frac{1}{\pi} \int_{4\pi} \Gamma(\Omega' \rightarrow \Omega) I(\tau, \Omega') \, d\Omega' \tag{2}$$

with the boundary condition

$$I(\tau_t, \Omega) = \int_{2\pi_-} f_s(\Omega', \Omega) |\mu'| I(\tau_t, \Omega') \, d\Omega' \tag{3}$$

for $\mu > 0$, where $f_s(\Omega', \Omega)$ is the bidirectional reflectance distribution function (BRDF) of background (e.g., soil) under the canopy, and $2\pi_-$ stands for the lower hemisphere. $G(\Omega)$ is a geometry factor discussed shortly, $\Gamma(\Omega' \rightarrow \Omega)$ is the area scattering function, and $h(\tau, \Omega, \Omega_0)$ is an empirical correction function to account for the variation of extinction coefficient [*Kuusk*, 1985; *Marshak*, 1989; *Nilson and Kuusk*, 1989]. Similar correction functions have been discussed in detail using geometric optical principles by *Jupp and Strahler* [1990]. Here we use *Nilson and Kuusk*'s [1989] formulation.

A statistical BRDF model [*Liang and Strahler*, 1994b] is used for the lower boundary condition because of its immediate accessibility:

$$f_s(\Omega_i, \Omega) = f_1(\Omega_i, \Omega) + f_2(\Omega_i, \Omega), \tag{4}$$

where

$$f_1(\Omega_i, \Omega) = b_0 + b_1\theta_i\theta \cos(\phi - \phi_i) + b_2\theta_i^2\theta^2 + b_3(\theta_i^2 + \theta^2)$$

$$f_2(\Omega_i, \Omega) = a_0 \exp[-a_1 \tan(\pi - \alpha)].$$

In these expressions, α is the phase angle between the incident direction (μ_i, ϕ_i) and the outgoing direction (μ, ϕ). In the hotspot direction $\alpha = \pi$, $f_2(\Omega_i, \Omega)$ reaches the maximum value. The value f_1 is the major component of the statistical BRDF model, and f_2 is significant only in the hotspot region. More detailed discussions about this model can be found elsewhere [*Liang and Strahler*, 1994b].

The function $G(\Omega)$ is the mean projection of a unit foliage area in the direction Ω [*Ross*, 1981; *Shultis and Myneni*, 1988], that is,

$$G(\Omega) = \frac{1}{2\pi}\int_{2\pi_-} g_l(\Omega_l)|\Omega_l \cdot \Omega|\, d\Omega_l \tag{5}$$

where $2\pi_+$ stands for the upper hemisphere. The function $g_l(\Omega_l)$ is the probability density of the distribution of the leaf normals with respect to the upper hemisphere. It is assumed that the zenith and azimuthal angles of the distribution of the leaf normals are independent and the distribution in the azimuth is uniform, that is, $g_l(\Omega_l) \equiv g_l(\mu_l)$. In all calculations the beta function [*Goel and Strebel*, 1984] is used for the leaf angle distribution.

In (2) the area scattering phase function $\Gamma(\Omega' \rightarrow \Omega)$, consisting of both diffuse and specular components, is defined as

$$\Gamma(\Omega' \rightarrow \Omega) = \Gamma_D(\Omega' \rightarrow \Omega) + \Gamma_{sp}(\Omega' \rightarrow \Omega). \tag{6}$$

We will find that $\Gamma(\Omega' \rightarrow \Omega)$ depends not only on the scattering angle between Ω' and Ω, but also on the absolute value of Ω' and Ω [*Shultis and Myneni*, 1988]. It is assumed that the diffuse scattering for the leaves follows the bi-Lambertian scattering model [*Ross*, 1981], given

$$\Gamma_D(\Omega' \rightarrow \Omega) = \frac{1}{2\pi}\int_{\Omega_+} g_l(\Omega_l)t_l\alpha'\alpha\, d\Omega_l - \frac{1}{2\pi}\int_{\Omega_-} g_l(\Omega_l)r_l\alpha'\alpha\, d\Omega_l. \tag{7}$$

Here Ω^+, Ω^- indicate that the Ω_l integration is over that portion of the 0–2π range for which the integrand is either positive or negative. In this model a fraction r_l of the intercepted energy is radiated in a cosine distribution about the leaf normal. Similarly, a fraction t_l is transmitted on the opposite side of the leaf. It is obvious that Ω^+, Ω^- is a part of the hemisphere for which $\pm\alpha'\alpha > 0$, $\alpha' = \Omega' \cdot \Omega_l$, and $\alpha = \Omega \cdot \Omega_l$.

The area phase function of specular component $\Gamma_{sp}(\Omega' \rightarrow \Omega)$ can be evaluated as [*Marshak*, 1989]

$$\Gamma_{sp}(\Omega' \rightarrow \Omega) = \tfrac{1}{8}\, g_l(\Omega_l^*)K(\kappa, \Omega' \cdot \Omega_l^*)F(n, \Omega' \cdot \Omega_l^*) \tag{8}$$

where $\Omega_l^* = \Omega_l^*(\Omega', \Omega)$ defines the direction of the appropriate leaf normal for specular scattering between incident and reflected rays [*Card*, 1987]. The term $F(n, \alpha')$ is the Fresnel reflectance, indicating the amount of specularly reflected energy for incident unpolarized radiance, and n is the wax refractive index of canopy leaves. The smoothing factor K is defined to account for the reduction in the amount of specularly reflected light due to the hair structure on the leaf surface [*Vanderbilt and Grant*, 1985; *Nilson and Kuusk*, 1989]. The argument $\kappa \geq 0$ characterizes the dimension of the hair on the leaf surface; $\kappa = 0.3$ has been used in this paper.

Radiation Field Decomposition

In order to characterize the hotspot phenomenon and handle multiple scattering effectively, the radiation field is decomposed into three components: unscattered radiance $I^0(\tau, \Omega)$, single-scattering radiance $I^1(\tau, \Omega)$, and multiple-scattering radiance $I^M(\tau, \Omega)$. The solutions for $I^0(\tau, \Omega)$ and $I^1(\tau, \Omega)$ are presented in our previous paper [*Liang and Strahler*, 1993a]. In the following we will focus on the calculation of multiple-scattering radiance.

3. Four-Stream Approximation for Multiple Scattering

Because the multiple-scattering radiance of the coupled medium cannot be explicitly calculated, we use instead a four-stream approximation. Our derivations begin with the atmosphere.

The Atmosphere Case

The azimuth-independent atmospheric radiative transfer equation for the total scattering radiance $I^a(\tau, \mu)$ is

$$\mu \frac{dI^a(\tau, \mu)}{d\tau} = I^a(\tau, \mu) - \frac{\omega_a}{2}\int_{-1}^{1} p_a(\mu, \mu')I^a(\tau, \mu')\, d\mu' - \frac{\omega_a}{4}F_0 p_a(\mu, -\mu_0) \cdot \exp\left(-\frac{\tau}{\mu_0}\right), \tag{9}$$

subject to the boundary condition:

$$\begin{aligned} I^a(0, \mu) &= 0 & \mu < 0 \\ I^a(\tau_a, \mu) &= I^c(\tau_a, \mu) & \mu > 0, \end{aligned} \tag{10}$$

where $I^c(\tau_a, \mu)$ is upwelling radiance from the canopy. The four-stream discrete ordinate solution to (9) at arbitrary level τ is given by *Chandrasekar* [1960] and *Liou* [1974]:

$$I^a(\tau, x) = \sum_{j=1}^{2} D_j(x) + Z(x) \exp\left(-\frac{\tau}{\mu_0}\right), \tag{11}$$

where

$$D_j(x) = [L_j W_j(x) \exp(-k_j\tau) + L_{-j} W_j(-x) \exp(k_j\tau)] \tag{12}$$

and $x = \pm\mu_1(0.3399810)$, $\pm\mu_2(0.8611363)$, and functions $W_j(x)$, $Z(x)$, and k_j are known [*Liou*, 1974]. $\mathbf{L}$ is a matrix of coefficients to be determined based on the boundary conditions. For the atmosphere the boundary condition is the final matrix equation for the coefficients $\mathbf{L}$ is

$$\mathbf{L} = \mathbf{A}^{-1}\mathbf{B}, \tag{13}$$

where the matrices $\mathbf{A}$ and $\mathbf{B}$ are given in Appendix A.

Now we need to find the solutions for arbitrary directions. Since the radiances at the four Gauss points have been determined, one natural way is to approximate the integration term in (9) using the four-point Gauss formula. Thus (1) for the scattering radiance becomes an ordinary differential equation:

$$\mu \frac{dI^a(\tau, \mu)}{d\tau} - I^a(\tau, \mu) = -\frac{\omega_a}{2} \sum_{l=-2}^{2} a_l p_a(\mu, \mu_l) \cdot \left[\sum_{j=1}^{2} D_j(\mu_l) + Z(\mu_l) \exp\left(-\frac{\tau}{\mu_0}\right)\right] - \frac{\omega_a}{4} F_0 p_a(\mu, -\mu_0) \exp\left(-\frac{\tau}{\mu_0}\right) \tag{14}$$

Solving the above equation, we have

$$I^a(\tau, \mu) = -\frac{\omega_a}{2} \sum_{l=-2}^{2} a_l p(\mu, \mu_l) \cdot \left[\sum_{j=1}^{2} E_j(\mu_l) - \frac{Z(\mu_l)\mu_0}{\mu + \mu_0} \exp\left(-\frac{\tau}{\mu_0}\right)\right] + \frac{\omega_a F_0 \mu_0 p(\mu, -\mu_0)}{4(\mu + \mu_0)} \exp\left(\frac{\tau}{\mu_0}\right) + C(\mu) \exp\left(\frac{\tau}{\mu}\right) \tag{15}$$

where

$$E_j(\mu_l) = \frac{-L_j W_j(\mu_l)}{k_j\mu + 1} \exp(-k_j\tau) + \frac{L_{-j} W_j(-\mu_l)}{k_j\mu - 1} \exp(k_j\tau)$$

for $\mu > 0$ and $\mu < 0$ but $|\mu| \neq \mu_0$. For the case of $\mu < 0$ and $|\mu| = \mu_0$ the radiance is calculated as

$$I^a(\tau, \mu) = -\frac{\omega_a}{2} \sum_{l=-2}^{2} a_l p(\mu, \mu_l) \left[\sum_{j=1}^{2} E_j(\mu_l) - \frac{Z(\mu_l)}{\mu}\tau\right] - \frac{\omega_a F_0}{4\mu} p_a(\mu, -\Omega_0)\tau + C(\mu) \exp\left(\frac{\tau}{\mu}\right). \tag{16}$$

The coefficient $C(\mu)$ can be easily determined using the boundary condition (10).

The Canopy Case

To calculate multiple scattering effectively by taking advantage of the existing four-stream formulation, further approximations need to be made. The canopy phase function is not rotationally invariant, but depends on both incident direction and outgoing direction. Our previous numerical results show that the multiple-scattering component is relatively insensitive to variation in azimuthal angle, since when the canopy becomes thicker optically, the photons will scatter more times before emerging from the canopy. Thus, we may accept the simplification that multiple scattering is independent of azimuth angle. Also, it seems that the multiple-scattering radiance distribution probably approaches the isotropic case. However, our results in a series of calculations show that the isotropic scattering function will cause large errors when there are a number of horizontal or vertical leaves in the canopy, as in the case of a planophile or erectophile canopy. Instead, a Henyey-Greenstein scattering phase function for multiple scattering and radiance independent of azimuth angles are assumed in this study. Although the Henyey-Greenstein function is an approximation to the real phase function and will cause some errors, the single-scattering radiance is still evaluated using the exact phase function. As a result, this formulation still can predict accurately the angular dependence of the reflectance.

The Henyey-Greenstein function is empirically related to biophysical parameters as follows. For spherically distributed leaves the area scattering phase function becomes [*Ross*, 1981]:

$$\Gamma(\Omega' \to \Omega) = \frac{\omega_c}{3\pi}(\sin\beta - \beta\cos\beta) + \frac{t_l}{\pi}\cos\beta$$

where $\beta = \cos^{-1}(\Omega \cdot \Omega')$, the angle between Ω and Ω'. The single scattering albedo of the canopy ω_c is defined as [*Myneni et al.*, 1990b]:

$$\omega_c = \omega_D + \omega_{sp} = r_l + t_l + K(\kappa, \Omega' \cdot \Omega_l) F(n, \Omega' \cdot \Omega_l), \tag{17}$$

which is an angular dependent quantity. In the calculation for multiple scattering a mean albedo is substituted:

$$\bar{\omega}_c = \frac{1}{2}\int_{-1}^{1} \omega_c(\mu)\, d\mu.$$

The phase function is defined by

$$p_c(\Omega' \to \Omega) = \frac{8\Gamma(\Omega' \to \Omega)}{\omega_c} = \frac{8}{3\pi}(\sin\beta - \beta\cos\beta) + \frac{8\alpha}{\pi}\cos\beta \tag{18}$$

where $\alpha = t_l/\omega_c$. In this case the canopy phase function depends on the phase angle only. Based on the above observation, we can approximate the above formula using the single term Henyey-Greenstein function. The asymmetry parameter g_c is directly related to the parameter α, and a simple function has been fitted using the least square principle from (18) and the Henyey-Greenstein function:

$$g_c = -0.2900 - 0.2478\alpha + 2.1653\alpha^2 - 1.1248\alpha^3 - 0.2059\alpha^4.$$

This functional relation is displayed in Figure 1. For the α ratio varying between 0.3 to 0.5, g_c is about 0 to -0.2. The negative value indicates strong backscattering.

For the unscattered radiance and single-scattering radiance the extinction coefficient has been modified to account for the hotspot effect. However, our derivation of the multiple-scattering component for the canopy is very similar to scattering radiance for the atmosphere. This implies that we do not consider the effect of finite leaf dimension for multiple scattering. The azimuth-independent radiative transfer equation is

$$\mu \frac{dI^c(\tau, \mu)}{d\tau} = I^c(\tau, \mu) - \frac{\omega_c}{2} \int_{-1}^{1} p_c(\mu, \mu') I^c(\tau, \mu')\, d\mu' - \frac{\omega_c}{4} F_0' p_c(\mu, -\mu_0) \exp\left(-\frac{\tau}{\mu_0}\right), \quad (19)$$

subject to the boundary conditions

$$I^c(\tau_a, \mu) = I^a(\tau_a, \mu) \qquad \mu < 0$$

$$I^c(\tau_0, \mu) = \frac{\pi}{0.52127} \sum_{j=1}^{2} a_j \mu_j r(-\mu_j, \mu) I^c(\tau_t, -\mu_j) + \mu_0 \pi F_0' r(-\mu_0, \mu) \exp\left(-\frac{\tau_c}{\mu_0}\right) \qquad \mu > 0 \quad (20)$$

where F_0' is the solar irradiance arriving at the top of the canopy, as attenuated by the atmosphere:

$$F_0' = F_0 \exp\left[-(\tau_a/\mu_0)\right].$$

The four-stream discrete-ordinate solution to (19) at arbitrary level τ is given by

$$I^c(\tau, x) = \sum_{j=1}^{2} D_j(x) + Z(x) \exp\left(-\frac{\tau}{\mu_0}\right). \quad (21)$$

$D_j(x)$ is defined in (12), and the corresponding **A** and **B** are given in Appendix B.

In a similar way we obtain the solutions for the arbitrary directions

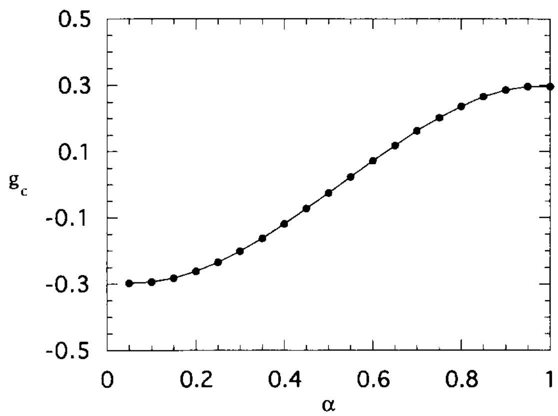

Figure 1. Illustration of the dependence of asymmetry parameter on the ratio of the leaf hemispheric transmittance to single scattering albedo.

$$I(\tau, \mu) = -\frac{\omega_c}{2} \sum_{l=-2}^{2} a_l p(\mu, \mu_l) \cdot \left[\sum_{j=1}^{2} E_j(\mu_l) - \frac{Z(\mu_l)\mu_0}{\mu + \mu_0} \exp\left(-\frac{\tau}{\mu_0}\right)\right] + \frac{\omega_c F_0 \mu_0 p_c(\mu, -\mu_0)}{4(\mu + \mu_0)} \exp\left(-\frac{\tau}{\mu_0}\right) + C(\mu) \exp\left(\frac{\tau}{\mu}\right) \quad (22)$$

for $\mu > 0$ and $\mu < 0$ but $|\mu| \neq \mu_0$. For the case of $\mu < 0$ and $|\mu| = \mu_0$ the radiance can be calculated:

$$I_c(\tau, \mu) = -\frac{\omega_c}{2} \sum_{l=-2}^{2} a_l p_c(\mu, \mu_l) \left[\sum_{j=1}^{2} E_j(\mu_l) - \frac{Z(\mu_l)}{\mu} \tau\right] - \frac{\omega_c F_0'}{4\mu} p_c(\mu, -\mu_0)\tau + C(\mu) \exp\left(\frac{\tau}{\mu}\right). \quad (23)$$

The coefficient $C(\mu)$ needs to be determined according to the boundary condition (20).

In the above, $I^c(\tau, \mu)$ is total scattered radiance. The multiple-scattering radiance is the difference between total scattering radiance and single-scattering radiance, as in *Liang and Strahler* [1993b].

4. Data Analysis

Since the radiative transfer equations of the atmosphere and canopy are coupled through the boundary conditions, solving the developed model requires an iteration process. Given the downward radiance of the atmosphere, the canopy radiation field is calculated by (22) and (23). Then the atmospheric radiation field is updated by (15) and (16). The cycling continues until convergence. Experiments show that if we use a pointwise convergence criterion [*Gerstl and Zardecki*, 1985], and it is set to 0.001, the iteration number is usually 2 or 3.

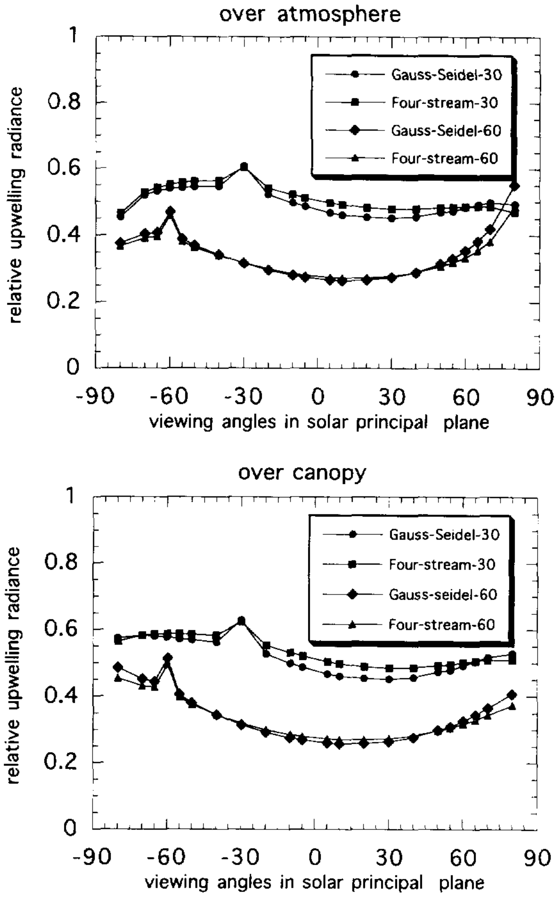

Figure 2. Comparisons of the present analytic model with Gauss-Seidel numerical model at two solar zenith angles 30° and 60°. The parameters are listed in section 4.

We assess the accuracy of the coupled four-stream model by comparing its output to that of our numerical code, which uses the Gauss-Seidel algorithm [*Liang and Strahler*, 1993a]. Since the numerical error is small, we assume that the numerical Gauss-Seidel solution is "exact." Because multiple scattering is not strong in the canopy in the visible region, our calculations are performed for the near-infrared region. Here is listed the "basic" parameter values in the following calculations, in which a cloudless atmosphere and a dense canopy of spherically distributed leaves are assumed.

r_l	0.52
t_l	0.40
τ_{ac}	0.10
τ_r	0.019
R_s	0.30
ω_a	0.92
g_a	0.65
LAI	3.0
LAD	uniform
k	0.08
n	1.45
θ_0	30°
ϕ_0	0°

R_s stands for the Lambertian surface reflectance. All the following calculations are implemented by setting $F_0 = 1$, and thus upwelling radiance is denoted as a relative quantity.

Note that radiance at the top of the atmosphere is from the coupled medium as a whole. Radiance at the top of the canopy is observed from the canopy illuminated by both sun and diffuse sky radiance.

Figure 2 illustrates relative upwelling radiance over both atmosphere and canopy in the principal plane with the standard parameter set in the above list. Overall, the four-stream model is quite accurate, especially at large solar zenith angles. When the viewing angle is not too large, say 60°, the present approximate model is very accurate at the top of the canopy. Overall, the relative error is about 5%.

Since the asymmetry parameter in the Henyey-Greenstein function is fitted from the spherical (uniform) canopy, a question naturally arises about the behavior of the model for the case of a canopy with a different leaf angle distribution. Figure 3 presents calculations for a planophile canopy. From this figure, we can see that when leaves are horizontally distributed, the accuracy does not decrease. Accurate results also demonstrate the case of the erectophile canopy, which has many vertical leaves. As compared to a spherical distribution, a planophile canopy increases upwelling radiance by about 5–10% because of the decreased chance that photons will penetrate downward.

Another experiment is to evaluate the accuracy of the present model with increased atmospheric turbity (Figure 4). It is interesting to note that at the top of the atmosphere the difference of radiance is smaller in the backscattering direc-

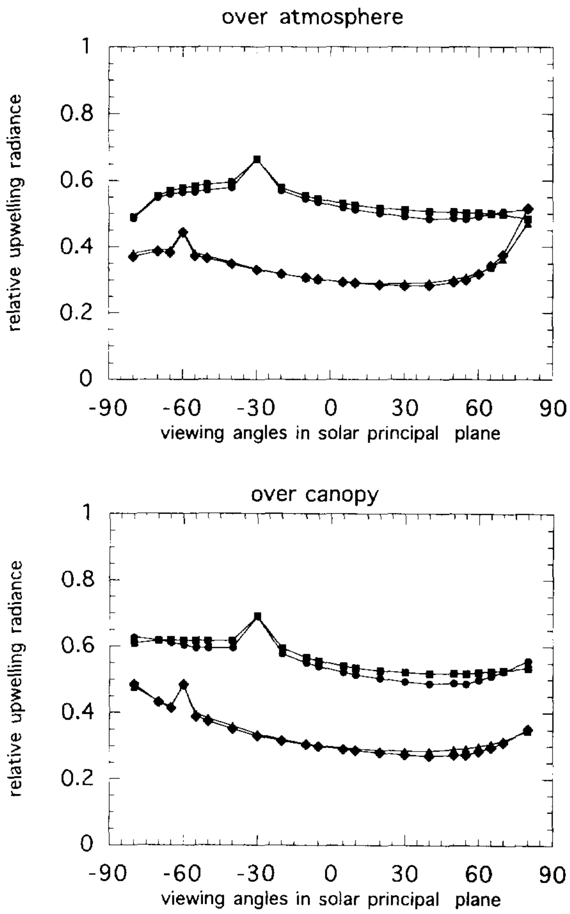

Figure 3. Validations of the present model with planophile canopy. The legend is the same as that in Figure 2.

tion and larger in the forward direction. Note that we are simulating very turbid atmospheres. In most cases the aerosol optical depth will be much smaller in the near-infrared region.

Figure 5 illustrates the (soil) surface BRDF effects in the near-infrared region where BRDF parameters are listed here:

b_0	0.04
b_1	0.017
b_2	0.002
b_3	0.063
a_0	0.251
a_1	0.418

With this set of BRDF parameters, the spherical albedo of the surface is 0.3. Figure 5a compares the four-stream model with the Gauss-Seidel model for the very dense canopy where multiple scattering becomes stronger. Even for the very dense canopy (LAI equals 6), the four-stream model is still very accurate. Theoretically, surface BRDF will increase the anisotropy of the canopy radiation field, especially for the thin canopy (low LAI). Since we have treated multiple scattering as azimuth-independent, it is necessary to also examine the angular characteristics of the radiation field of the sparse canopy. Figure 5b presents results in the 225°–45° azimuth plane for a canopy with low leaf density (LAI equals 1.5). The accuracy is quite satisfactory, probably because when the canopy becomes sparse, single-scattering dominates. Thus, approximating multiple scattering does not seriously decrease the accuracy of the model.

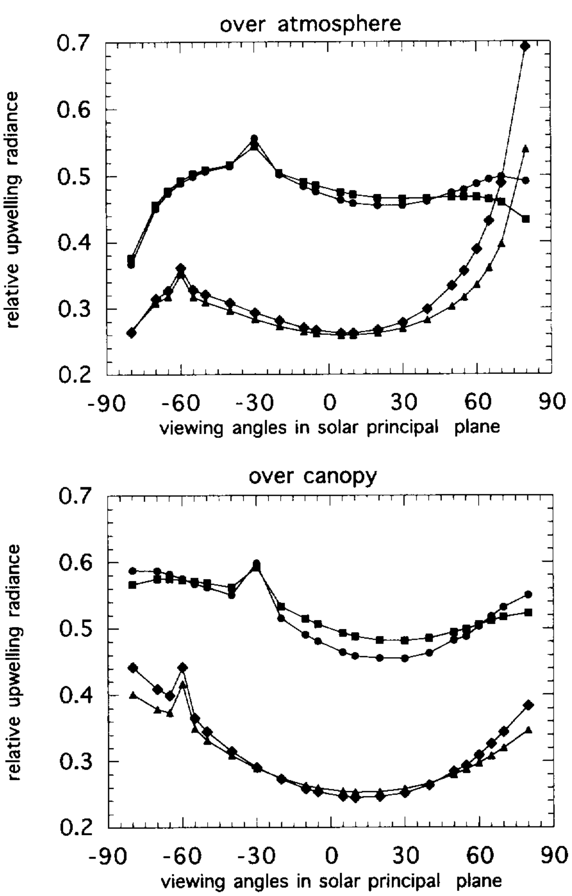

Figure 4a. Evaluations of the analytic model with very turbid atmosphere. Aerosol optical depth is 0.3. The legend is the same as those in Figure 2.

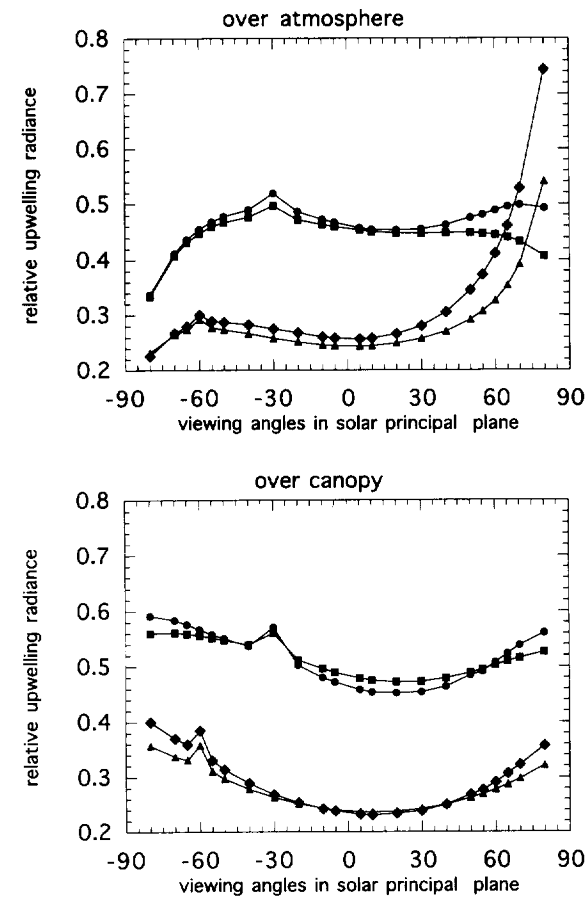

Figure 4b. Evaluations of the analytic model with very turbid atmosphere. Aerosol optical depth is 0.5. The legend is the same as that in Figure 2.

Note that the reflectance of the surface underlying the canopy may be smaller than 0.3 in many situations because of its high moisture. In that case the interaction between the canopy and the underlying surface is smaller, and the approximate model may have higher accuracy.

In most parametric canopy models, no explicit sky radiance distribution has been considered. One of the common practices is the use of the following formula for the upwelling radiance at the top of the canopy due to sky radiance $I^s(\Omega)$:

$$I(\Omega) = \frac{1}{\pi} \int_0^{2\pi^-} I^s(\Omega')R(\Omega', \Omega)|\mu'| \, d\Omega'$$

where $R(\Omega', \Omega)$ is the canopy bidirectional reflectance at the viewing direction Ω given the diffuse illumination at the directions Ω'. If Gaussian quadrature is used the above integration can be replaced by the sum over M (zenith angle) by N (azimuth angle) discrete directions. Obviously this requires MN calculations of the parametric canopy model, and thus the advantage of quick calculation may be lost. Another problem is that we usually do not know the sky radiance a priori because it is a function of the surface reflectance. Both of these concerns are addressed by a model that couples the atmosphere and the canopy.

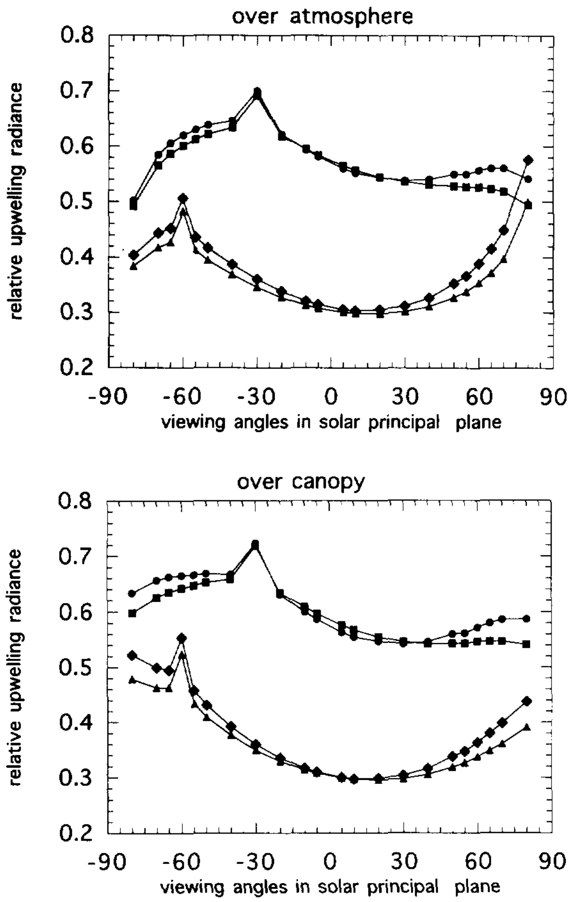

Figure 5a. Evaluations of the effects of surface BRDF on the present model at an LAI of 6.0. The legend is the same as that in Figure 2.

5. Brief Summary and Discussion

An approximate radiative transfer model of the coupled atmosphere and canopy has been described in this paper. The multiple-scattering component is approximated by the four-stream approach, while other components are exactly calculated. This model considers both internal scattering and leaf specular reflectance, and the bidirectional reflectance distribution function (BRDF) of the soil surface underneath the canopy is also incorporated in the formulation. Because an empirical formula relates canopy phase function used for multiple scattering to biophysical parameters of the canopy, all parameters that describe the canopy model have very clear physical meanings.

The numerical Gauss-Seidel algorithm is used as a standard of comparison for this present model. The results indicate that the four-stream approximate model is very accurate, even for very dense canopies and turbid atmospheres in which multiple scattering dominates.

Since the atmospheric path radiance and sky radiance are well accounted for in this model, it is well suited to the analysis of airborne and spaceborne multiangle remotely sensed imagery such as that of ASAS and that expected for MISR. A further research activity is to retrieve biophysical parameters of homogeneous canopies from multiangle remotely sensed data using this model. However, since most natural plant canopies are quite heterogenous, approximate three-dimensional radiative transfer models also need to be developed. The plane-parallel case we discuss here is a necessary step in that development.

Because the surface and atmosphere interact through multiple scattering, only a coupled model can accurately describe their interaction and its influence on top-of-the-atmosphere radiances. However, the question arises as to whether or not the added complexity of a coupled model is necessary. Simulation of above-canopy and above-atmosphere radiances shows that at shorter wavelengths, atmospheric scattering provides a strong signal at the top of the atmosphere [*Liang and Strahler*, 1993a]. However, at longer wavelengths, the contribution is not so large. What is important in this situation is the effect of the atmosphere on within-canopy multiple scattering, which is still significant [*Liang and Strahler*, 1993a]. We have not attempted to assess the errors inherent in calculating radiances for the cases of isotropic diffuse radiance, no diffuse radiance, or atmospheric correction based on the assumption of a Lambertian lower boundary (isotropic canopy-soil reflectance), since in our opinion these are unrealistic assumptions. However, a careful study of the effect of these assumptions on top-of-the-atmosphere radiances would identify the conditions under which noncoupled models are clearly inappropriate.

A few recent papers confirm the importance of coupling the surface and atmosphere in directional reflectance modeling. *Myneni et al.* [1992, 1993] have coupled both one- and three-dimensional radiative transfer canopy models to atmo-

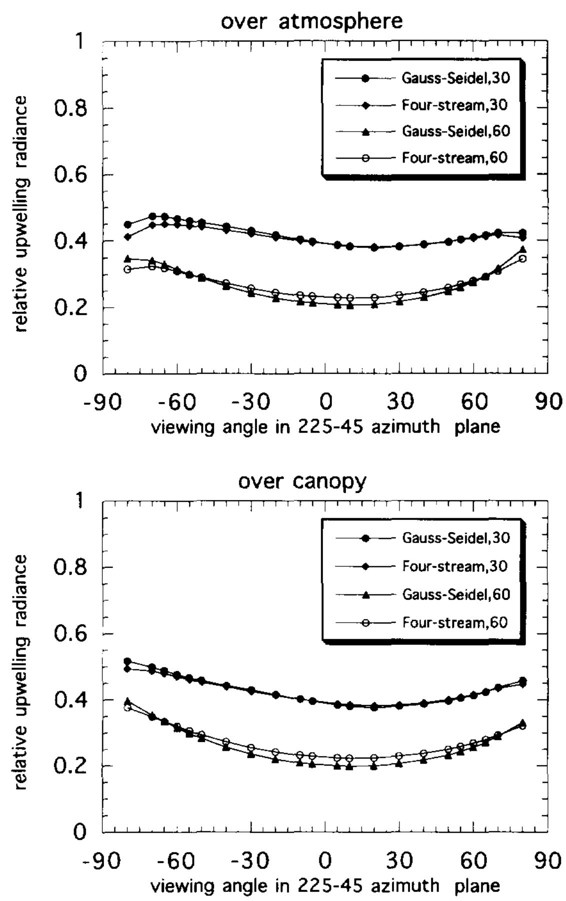

Figure 5b. Evaluations of the effects of surface BRDF on the present model at an LAI of 1.5.

spheric radiative transfer models, showing that the atmosphere adds significant path radiance to surface radiance at red wavelengths, while the atmosphere significantly attenuates surface radiance at near-infrared wavelengths. Adjacency effects, which are produced by surface-atmosphere interaction, are also reproduced well by the coupled model. *Rahman et al.* [1993a] coupled an atmospheric model with the surface reflectance model of *Verstraete et al.* [1990] and *Pinty et al.* [1990] using a multiple reflectance parameter that depends on the proportions of direct and diffuse irradiance. Using this model, they demonstrated the potential of retrieving canopy optical and structural parameters from simulated advanced very high resolution radiometer (AVHRR) top-of-atmosphere radiances. These authors also provide a semiempirical BRDF model that is applied to AVHRR data from the North Africa desert with good results [*Rahman et al.*, 1993b]. For some test datasets the authors adjusted observed reflectances for the smoothing of the BRDF that is produced by diffuse illumination, confirming the importance of coupling the atmosphere and canopy.

The four-stream model described here can be considered a midpoint between the two approaches presented by *Myneni et al.* [1992, 1993] and *Rahman et al.* [1993a, b]. It provides a more complete description of the physics of multiple scattering than that of Rahman et al., but avoids the necessity of finding numerical solutions. As such, it should be well suited to a number of applications in remote sensing, both for forward and inverse modeling.

Appendix A: Atmosphere

The following are coefficient matrices for the equation $\mathbf{L} = \mathbf{A}^{-1}\mathbf{B}$ in the case of the atmosphere:

$$\mathbf{A} = \begin{bmatrix} W_1(-\mu_1) & W_2(-\mu_1) & W_1(\mu_1) & W_2(\mu_1) \\ W_1(-\mu_2) & W_2(-\mu_2) & W_1(\mu_2) & W_2(\mu_2) \\ W_1(\mu_1)t_1 & W_2(\mu_1)t_2 & \dfrac{W_1(-\mu_1)}{t_1} & \dfrac{W_2(-\mu_1)}{t_2} \\ W_1(\mu_2)t_1 & W_2(\mu_2)t_2 & \dfrac{W_1(-\mu_2)}{t_1} & \dfrac{W_2(-\mu_2)}{t_2} \end{bmatrix}$$

$$\mathbf{B} = [b_1, b_2, b_{-1}, b_{-2}]^t,$$

where $[\quad]^t$ denotes the matrix transpose, and the corresponding parameters are defined as

$$t_1 = \exp(-k_1\tau_a)$$
$$t_2 = \exp(-k_2\tau_a)$$
$$b_1 = -Z(-\mu_1)$$
$$b_2 = -Z(-\mu_2)$$
$$b_{-1} = I^c(\mu_1) - Z(\mu_1)\exp(-\tau_a/\mu_0)$$
$$b_{-2} = I^c(\mu_2) - Z(\mu_2)\exp(-\tau_a/\mu_0).$$

Matrix $\mathbf{A}^{-1}$ can be calculated by any numerical calculation package. However, to avoid an unnecessary iteration process, an explicit formula for finding $\mathbf{L}$ can be found in our previous paper [*Liang and Strahler*, 1994a].

Appendix B: Canopy

$$\mathbf{A} = \begin{bmatrix} W_1(-\mu_1) & W_2(-\mu_1) & W_1(\mu_1) & W_2(\mu_1) \\ W_1(-\mu_2) & W_2(-\mu_2) & W_1(\mu_2) & W_2(\mu_2) \\ [\alpha_{-1}(\mu_1) - W_1(\mu_1)]t_1 & [\alpha_{-2}(\mu_1) - W_2(\mu_1)]t_2 & \dfrac{\alpha_1(\mu_1) - W_1(-\mu_1)}{t_1} & \dfrac{\alpha_2(\mu_1) - W_2(-\mu_1)}{t_2} \\ [\alpha_{-1}(\mu_2) - W_1(\mu_2)]t_1 & [\alpha_{-2}(\mu_2) - W_2(\mu_2)]t_2 & \dfrac{\alpha_1(\mu_2) - W_1(-\mu_2)}{t_1} & \dfrac{\alpha_2(\mu_2) - W_2(-\mu_2)}{t_2} \end{bmatrix}$$

$$\mathbf{B} = [b_1, b_2, b_{-1}, b_{-2}]^t$$

where

$$\alpha_{-1}(x) = \frac{\pi}{0.52127}\sum_{j=1}^{2} a_j\mu_j r(-\mu_j, x)W_1(-\mu_j)$$

$$\alpha_{-2}(x) = \frac{\pi}{0.52127}\sum_{j=1}^{2} a_j\mu_j r(-\mu_j, x)W_2(-\mu_j)$$

$$\alpha_1(x) = \frac{\pi}{0.52127}\sum_{j=1}^{2} a_j\mu_j r(-\mu_j, x)W_1(\mu_j)$$

$$\alpha_2(x) = \frac{\pi}{0.52127}\sum_{j=1}^{2} a_j\mu_j r(-\mu_j, x)W_2(\mu_j)$$

$$t_1 = \exp(-k_1\tau_c)$$

$$t_2 = \exp(-k_2\tau_c)$$

$$b_1 = I^a(-\mu_1) - Z(-\mu_1)$$

$$b_2 = I^a(-\mu_2) - Z(-\mu_2)$$

$$b_{-1} = -\left[\frac{\pi}{0.52127}\sum_{j=1}^{2} a_j\mu_j r(-\mu_j, \mu_1)Z(-\mu_j)\right.$$

$$+ \mu_0 \pi F_0' r(-\mu_0, \mu_1) - Z(\mu_1) \Bigg] \exp\left(-\frac{\tau_c}{\mu_0}\right)$$

$$b_{-2} = -\left[\frac{\pi}{0.52127} \sum_{j=1}^{2} a_j \mu_j r(-\mu_j, \mu_2) Z(-\mu_j)\right.$$

$$\left. + \mu_0 \pi F_0' r(-\mu_0, \mu_2) - Z(\mu_2)\right] \exp\left(-\frac{\tau_c}{\mu_0}\right).$$

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